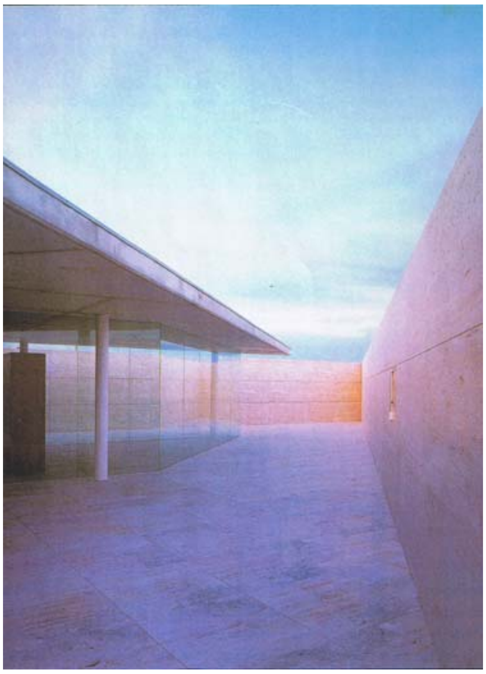
A light well

Pedro Martín García



Perimetral walls from CDER building in Majorca, Spain
From "Alberto Campo Baeza: Works and Projects", compiled by Antonio Pizza, published by Gustavo Gili, Barcelona, 1999

On November, 1998, Alberto Campo showed me some photographs of the interior corner of the perimetral walls of CDER building (Mallorca). The travertine walls, oriented in that way that direct light from dawn incided within the angle, were illuminated in a pale blue color except in the corner, where light concentrated strongly and showed an intense orange tone.

"Is there something that casts shadows over the non-illuminated portion of the wall? Perhaps they are not allowing diffuse bluish light to be scattered on the corner". No, there is no such thing. "Perhaps diffuse light from inner glass walls is colouring the walls blue?" Maybe, but it will shade the corner too. "Its it a photographic effect?" Well, colors had been probably somehow saturated, but in this other picture the effect remains unaltered —concentration remains as intense as before—. "I've seen this effect here, and it's real".

By then I was developing some studies about light scattering in light control operations like skylights or spanish *transparentes*. And I decided to put myself at work.

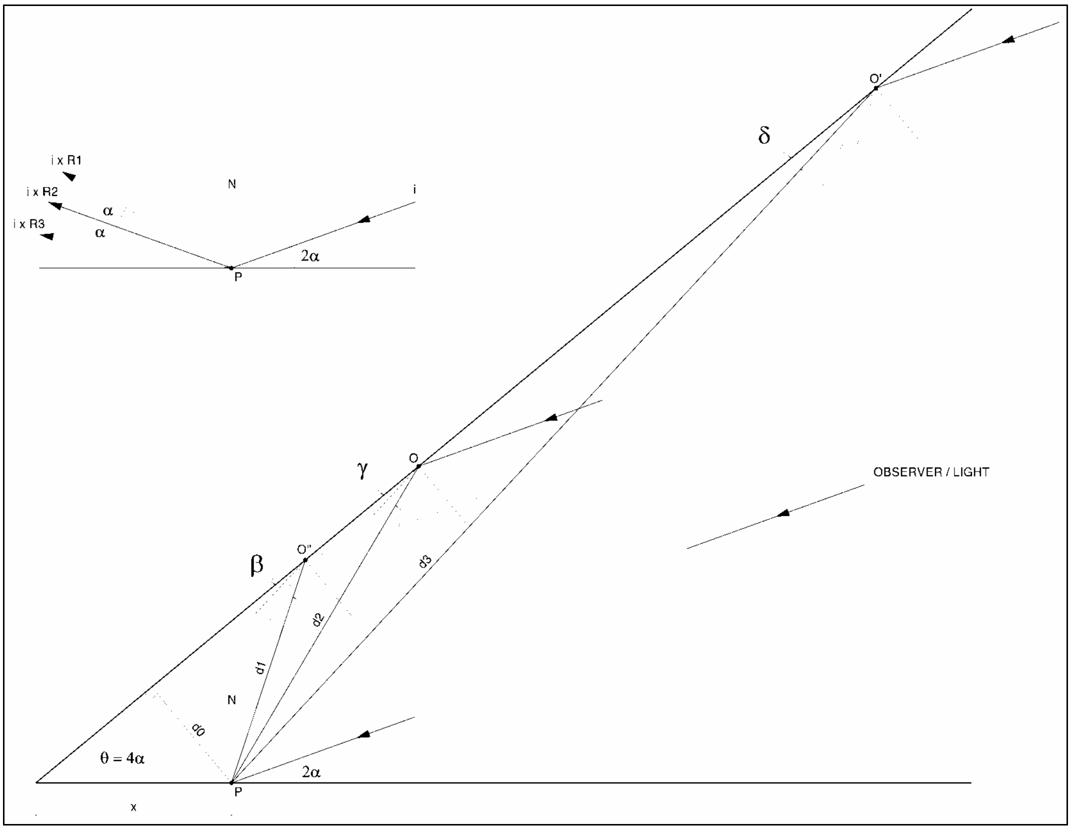
Where does come this effect from? For now, let's leave colors aside. The fact is that in any angle conformed by two surfaces of any material, illuminated directly by light in a direction within the angle, scattered diffuse light takes care of intensifying the illuminance (and of course perceived light from the observer) towards the corner following a exponential function. This effect appears in any angle (even greater than 90 degrees) and with any material, but the effect is highly increased if:

- surfaces are *lambertian*
- angle is steep, especially smaller than 45 degrees
- light and observer are placed in the bisector angle

And these conditions are present on CDER's angle.

Let's go to the math.

The next graphic shows an angle between two surfaces. We assume first that light incides exactly via the bisector angle, and second that an observer is looking through the same direction.



Let's consider a point P. It receives light from the bisector angle, forming 2α with the surface. Then, if we assume that point P not only receives direct light, but light from other points on the opposite surface, the general illumination equation in point P should be written like this:

$$\mathbf{i_P = R_r\,(i_d + [(i_d\,R_1) / d_1^2] + [(i_d\,R_2) / d_2^2] + [(i_d\,R_3) / d_3^2])}$$

This equation has the form $i_p = f\,(1/x^2)$, so to say, an hyperbola. In this formula $\mathbf{i_p}$ stands for total illumination perceived, while $\mathbf{i_d}$ shows direct illumination received on point P. $\mathbf{d_1}$, $\mathbf{d_2}$ and $\mathbf{d_3}$ are the distances to the point P from the points on the other surface, as shown in the graphic; its values can be defined as f(x), **x** being the distance from P to the corner, and they can be geometrically obtained from trigonometry:

$d_0 = x \sin 4\alpha = 4\,x \sin\alpha \cos\alpha\,(\cos^2\alpha - \sin^2\alpha)$

$\beta = 2\alpha + \alpha = 3\alpha;\ \sin\beta = 3\sin\alpha - 4\sin^3\alpha$

$\gamma = 2\alpha;\ \sin\gamma = 2\sin\alpha\cos\alpha$

$\delta = 2\alpha - \alpha = \alpha;\ \sin\delta = \sin\alpha$

$\mathbf{d_1} = d_0 / \sin\beta = 4\,x \sin\alpha \cos\alpha\,(\cos^2\alpha - \sin^2\alpha) / (3\sin\alpha - 4\sin^3\alpha)$

$\mathbf{d_2} = d_0 / \sin\gamma = 2\,x\,(\cos^2\alpha - \sin^2\alpha)$

$\mathbf{d_3} = d_0 / \sin\delta = 4\,x \sin\alpha \cos\alpha\,(\cos^2\alpha - \sin^2\alpha)$

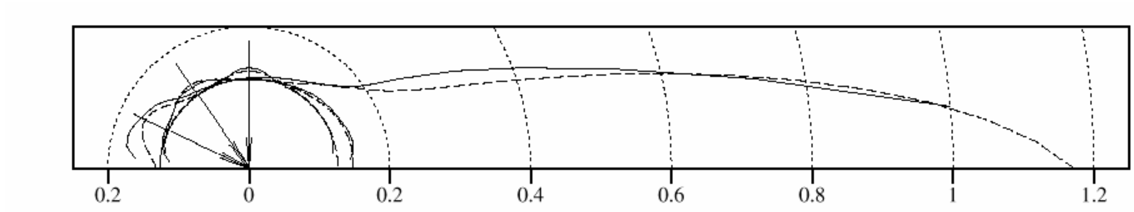
Now, we had to know the values of $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$. But, why do we use three values instead of the usual unique reflection parameter? Well, that's probably the key of the problem.

Real surfaces don't usually behave as perfect mirrors. In fact, when a ray of light incides on a surface, it is scattered not only in the main reflectance angle (same as incident), but usually they do it in a lot of directions, even back to the light source —*retro-reflection*—. When a surface scatters more than 50% of incident light in directions different from the main reflection angle, it is called a *lambertian surface*[i].

Usual pure lambertian surfaces are chalk, white paper, matte white paint or even moon's surface. In fact, all natural material are in some way lambertian.

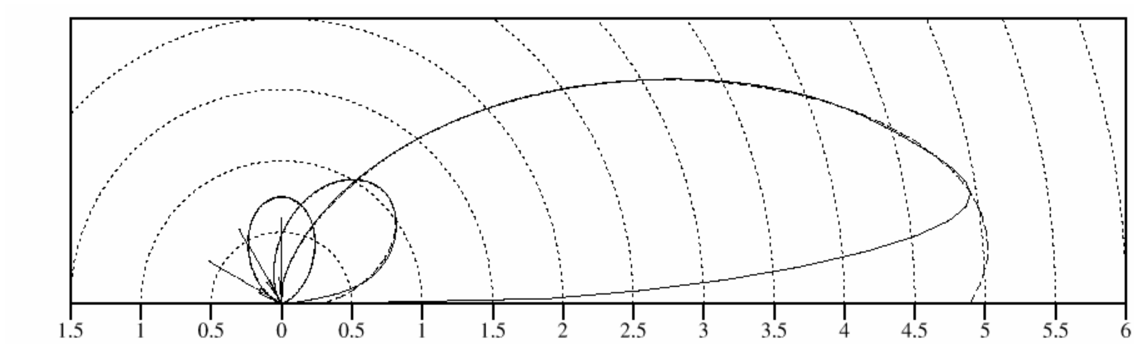
And, of course, travertine is.

The following diagram shows the BRDF[ii] data (in the plane of incidence for $2\alpha = 0^o$, 35º and 65º at 550nm) for a lambertian surface (in this case, matte blue paint)[iii] . It shows lambertian reflection (the approximate circle around the incident point) and retro-reflection (small peaks back to the source).



But there's something more: as long as we reduce the incident angle, the amount of reflection increases up to 10 times the usual value. This can be useful.

In the case of more reflective material, such as roughened aluminum, reflection increases too at grazing angles, but the relation between them and steep the angle's values are only (1 to 6,5), as shows the following diagram:



Some more, we do not have barely retro – reflection. Is this important? Of course it is. The light we receive from a lambertian surface may come back to the light source, but here it can't. So, the only light we can receive from a point in a reflective surface would come from successive mirror-like reflections within the angle. This process involves a long distance for the reflected rays (light intensity decreases with the inverse of the square of the distance); in each bounce we lose at least 10 percent of the energy, so the light reaches to us notably reduced. We will seethe angle darkened, although what little light that comes out shows us a perfect mirror reflection.

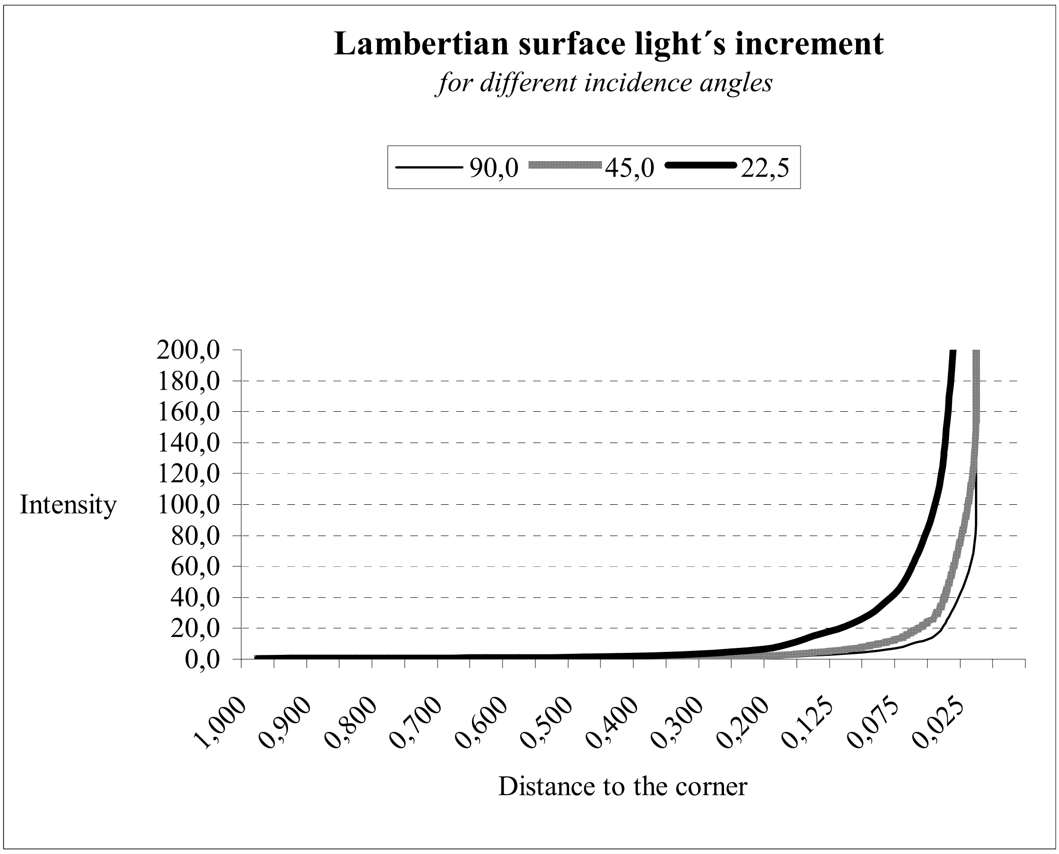
On the other side, light coming to us by retro-reflection from a lambertian surface comes only before one bounce (or two if it is coming from the other surface), so it is not greatly reduced in intensity.

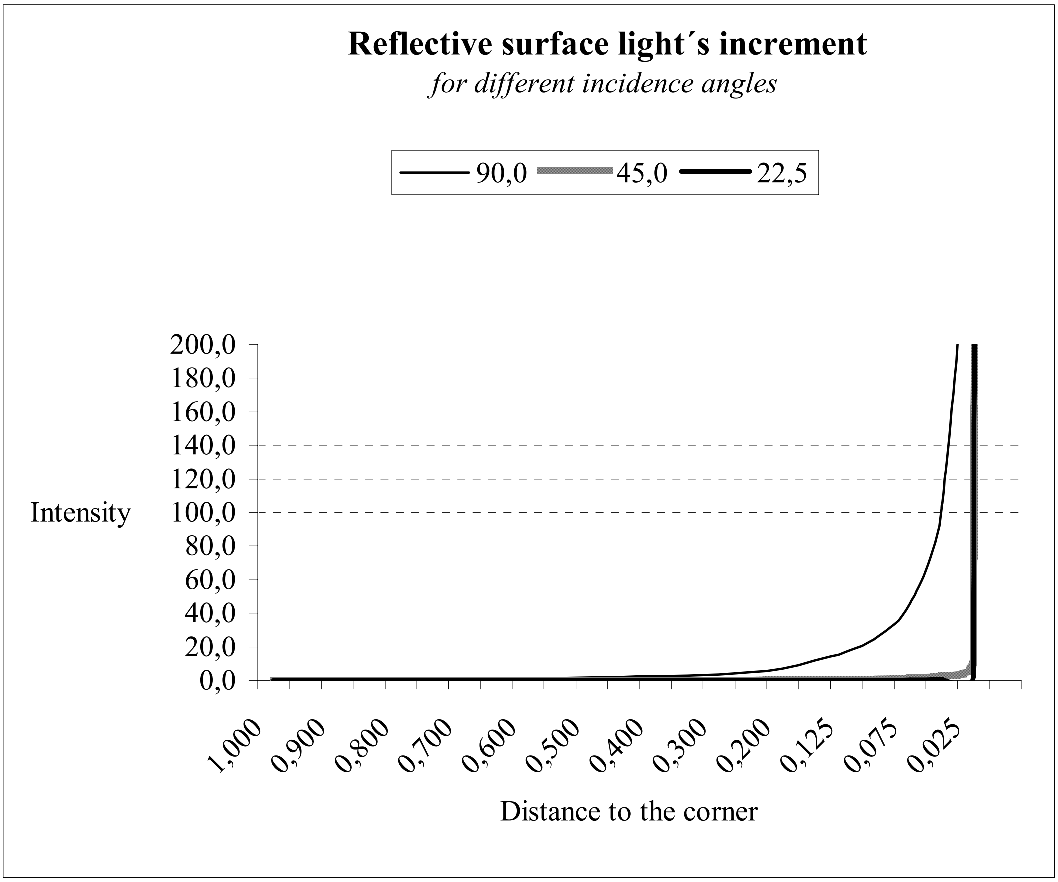
For our equation, we'll make scattering easier to manage taking only three values R_1, R_2 and R_3 as reflection coefficients; R_1 for angle – α respect specular reflection angle, R_2 for the reflection angle itself, and R_3 for angle + α. Finally, we'll use retro–reflection as a factor R_r that affects the whole illumination value. The next table shows values for R_1, R_2, R_3 and R_r extracted from the previous graphics.

BDRF data of typical surfaces

	Lambertian surface			Retro - reflection	Non - lambertian surface	
Incident angle	*R 1*	*R 2*	*R 3*		*R 1*	*R 2*
90	0,15	0,17	0,15	0,15	0,00	0,75
55	0,18	0,17	0,16	0,15	0,90	0,90
45	0,33	0,22	0,17	0,16	2,27	1,77
25	0,62	0,31	0,19	0,19	5,00	3,50

If we now make the necessary substitutions in the formula of general illumination to obtain values for the rise of intensity in function of the distance to the angle, we obtain the real expressions for typical incidence angles (90, 45 and 22,5º) for two kind of surfaces, one of them purely lambertian (as travertine), and the other more reflective (rough aluminum). The following tables show the results.





In these graphics we can see that the rise of intensity towards the corner for near to 45 degree incidence angles (that´s it, 90 degree surface angles) is barely perceivable for both kinds of surfaces, although the average value of lambertian material is higher than it is for the reflective material. But when the angle value decreases, the lambertian's surface increments quickly exceed those from the reflective surface. At 22,5 degree incidence angle —such as found at the CDER building— the increment is much higher than the equivalent for aluminum —almost null—, and as long as we reach zero, the difference increases radically. As the curves show, only when we look at reflective surface from 90 degree do we found its values to be higher.

Well, what do I had then? Just a lot of numbers and equations, although they seemed to be based on solid foundations. But I needed another evidence. A simple example of a small angle formed with lambertian material such chalk or paper.

Paper, that's it.

So I folded a blank piece of paper, and oriented it towards direct light from sun.

And there it was. I've captured it.

Then I folded that piece of light once more, to avoid it to escape, put it on my pocket and went to talk to Alberto.

January 1999

---

[i] Johann Heinrich Lambert (1728 – 1777) studied light scattering and wrote *Photometria* (1760). He gave birth to the main reflection law, Lambert's Law.

[ii] BDRF stands for *bi-directional reflectance distribution function*.

[iii] BDRF graphics taken from Eric P. F. Lafortune, Sing-Choong Foo, Kenneth E. Torrance, Donald P. Greenberg: "Non-Linear Approximation of Reflectance Functions", Program of Computer Graphics, Cornell University (from Cornell University WWW pages).